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MANAGEMENT OF THE RURAL TOURISM CLUSTER BASED ON CASH FLOWS' ECONOMIC-MATHEMATICAL MODE

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Abstract
The management of a rural tourism cluster based on an economic-mathematical model stipulates the use of mathematical modeling methods to optimize the economic parameters of the cluster's activity. Therefore, the practical aspects of the application of a system approach to the development and improvement of the management system in a rural tourism cluster are substantiated. The article aims to develop an economic-mathematical model for the optimization of the management of the rural tourism cluster parameters. The model uses the Kalman filter and deterministic feedback to evaluate the state of the dynamic system based on observations. The developed models and the management optimization algorithm are synthesized in the automatized information-analytical management subsystem. The typical list of business processes in the activity of a rural tourism cluster is characterized to specify managerial actions and improve their efficiency and performance. The ways of the algorithm's integration into the automatized information-analytical management subsystem are outlined. Overall, the article offers the method of optimization of rural tourism cluster management based on mathematical modeling and management optimization.

Keywords: *management, rural tourism cluster, economic-mathematical model, cash flows.*
JEL Codes: *: L83, O13, O33, D20*.

Introduction

The tourist cluster is a system of intensive production-technological and information interaction of tourist enterprises, providers of basic and additional services based on the creation of a joint tourist product. Rural tourism clusters include groups of enterprises geographically concentrated within rural areas, which share tourist resources, specialized tourist infrastructure, local labor markets, and carry out joint marketing and advertising activities. The overall structure of a tourism cluster consists of four units.

They include tourism service providers, material suppliers, tourists, and various support systems.

Rural tourism cluster management is a functioning system that consists of rational relationships of elements. The importance of a systems approach is aimed at obtaining a certain useful management result. The purpose of rural tourism cluster management as a system is survival and development. Survival is possible only with a profit that is sufficient to maintain the appropriate level of solvency and income. Its development is possible in the presence of capital: owners and creditors. This in turn requires that the expected long-term profit exceeds the norm of alternative profitability.

The research is relevant since the offered economic-mathematical model of cash flows is developed to reflect the relationships between different components of the rural tourism cluster such as accommodation, transportation, and attractions. The model aims to determine the optimal combination of these components to maximize the total income and profitability of the cluster.

The novelty of the research lies in the fact that the value of a rural tourism cluster is not only an indicator of the activity of managers but also an object of management. The quality of the management system significantly depends on the information that can be obtained by evaluating and processing the values of the parameters of its economic activity, and the solution of the problem of management synthesis by elaborating the economic-mathematical model of the rural tourism cluster.

Literature review

Rural tourism is currently among the most perspective directions of the tourism industry development. It becomes the ground for boosting the local business policy. The issues are addressed by researchers from the viewpoint of various approaches and attraction of adjoining economic activities to these system transformations of businesses. Insurance and investment attraction are the major sources for developing the activity in the industry, which is verified in the studies of a range of researchers (M. Dziamulych et al., 2021, A. Kucher et al., 2019, I. Kondrat et al., 2019, A. Sakhno et al., 2019, O. Kovalova et al., 2022).

The research by L. Wu et al. (2019) focuses on the application of the VIKOR method in financing risk assessment of rural tourism projects under interval-valued intuitionistic fuzzy environment. The study uses the VIKOR method as a multiple attribute decision-making (MADM) technique to evaluate and rank the financing risk of rural tourism projects. The study proposes an interval-valued intuitionistic fuzzy VIKOR method that considers the uncertainty and vagueness of the decision-making process. The proposed method is applied to a case study of financing risk assessment of rural tourism projects in China.

P. Trivellas et al. (2015) offer an economic-mathematical model for evaluating the sustainability of rural tourism, taking into account the economic, social, and environmental factors influencing the sector.

N. Vdovenko and N. Sydorova (2020) propose an economic-mathematical model for managing rural tourism clusters that takes into account the economic and social factors influencing the cluster's performance.

R. Chobanova and I. Yotsov (2014) provide an overview of the role of mathematical modeling in rural tourism development, highlighting the benefits of using mathematical models to optimize the performance of tourism clusters.

The other researchers address the issues of developing this type of economic activity based on the development and attraction of land resources to improve the tourism infrastructure (T. Kalashnikova et al., 2019, A. Popov et al., 2019, R. Sodoma et al., 2021). Domestic researchers expand the studies on the opportunities and capacities of the tourism infrastructure development in Ukraine (I. Kinash et al., 2019, H. Horban et al., 2020, Yu. Kovalchuk et al., 2020). E. Fedorova and I. Vasilyeva (2019) offer a multi-criteria decision-making model for developing rural tourism clusters that considers factors such as the attractiveness of the tourism product, the

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quality of the tourism infrastructure, and the availability of financial resources.

The article by M. Viganò et al. (2018) proposes a cash flow analysis tool for small and medium-sized enterprises (SMEs) in the rural tourism sector to help with business planning and control.

Regarding rural tourism development in Ukraine, H. Zelinska et al. (2021), O. Karyy et al. (2021) examine it from the viewpoint of creating the recreational atmosphere. Korkuna O.I. et al. (2020) argue that rural tourism is the industry that includes the provision of accommodation or a place for a tourist to stay outdoors (in the forest, mountains, village, etc.). A. Yakymchuk et al. (2021), O. Olshanska et al. (2021) verify the dependence between the growing income of a tourist and expenditures on tourism. The authors analyze the dynamics of the GDP in Ukraine and the share of income from the provision of tourism services. Y. Danko (2018) argues that rural tourism is the type of entrepreneurial activity that uses private ownership for the enrichment of the locals. I. Hryhoruk et al. (2021) and T. Skrypko et al. (2020) examine rural tourism from the viewpoint of the hotel industry development. M. M. Bil (2018) addresses the tourism mobility of the population, its nature, global trends, relationship with migration processes, and impact on the development of rural tourism.

However, these studies lack detailed research of rural tourism development management, especially with regard to the use of mathematical approaches, which is the subject of this paper.

Methodical approach

The estimate of the expected long-term profit of the rural tourism cluster can be determined using the procedure of discounting cash flows of the rural tourism cluster (*Discounted Cash Flow – DCF*). The first such approximation corresponds to the price of the rural tourism cluster, which, due to the aggregate, can act as a target management function. Price as an integral criterion of management efficiency makes it possible to assess all types of organizational change (rather than individual activities). This allows giving all business processes a targeted nature and managing the facility as a whole system. The "rural tourism cluster" system potentially seeks to get as close as possible (achieve) to the target management function, ie to maximize its price.

The essential parts of the rural tourism cluster system are assets, liabilities and employees. All assets of the rural tourism cluster, with the exception of goodwill, reflect tangible and intangible resources that are directly involved in the production process.

Liabilities reflect the sources of asset formation, ie the capital consumed by the rural tourism cluster and the rights of ownership or of capital, staff with his knowledge and experience, as well as corporate spirit. The control links of the system elements are the functions performed by the management system to achieve the set goals.

The activity of any system is characterized by a system effect on the principle of emergence, which reflects the quality of the subsystems as a whole, ie the efficiency of the elements of the system and the effectiveness of control relationships. The system effect can be determined by comparing the total cost of system elements and the cost of the system as a whole by its objective function. The system effect is the difference between the income value of the rural tourism cluster (price) and the cost of its elements (liquidation value). This figure corresponds to the concept of goodwill. The systemic effect can be both positive and negative. A negative value of the system effect indicates the inefficiency of the system management.

The price of a rural tourism cluster is an important comprehensive assessment that adequately reflects the economic efficiency, its financial position and prospects. It is very sensitive to any changes in the environment, both internal and external: reduced profitability, deteriorating solvency, increased investment risk, and reduced competitiveness. All this leads to a decrease in the income value of the rural tourism cluster. Today the indicator of the income value of the rural tourism cluster has not been properly applied, although almost all management decisions (from changing the lease of land to full ownership of it, restructuring) are made in order to increase the profitable value of the rural tourism cluster.

The price of a rural tourism cluster is not only an indicator of managers' activity, but also an object of management. In the world experience, factors concerning the levers of influence on the price of the rural tourism cluster can be noted: the cost of land, assessment of capital structure, the efficiency of invested capital.

Methods of estimating the price of a rural tourism cluster depend on the situation: the nature of the market in which it operates, the specifics of the rural tourism cluster itself, the quality of information flows, and so on. Unlike independent experts who evaluate the activities of the rural tourism cluster, the management staff has the opportunity to use methods to more accurately diagnose the impact of growth factors. It is worth noting the method of cash flows, which is based on the consolidation by discounting to a certain point in time future cash flows. They practically remain at the disposal of the rural tourism cluster for each period, which allows for the establishment of the dependence of the amount of free funds to the extent to which the sources of replenishment will be adjusted for changes in working capital. Based on this thesis, a model of cash flows of the rural tourism cluster is built.

This model of a rural tourism cluster is designed to describe the functional relationship between the system of key performance indicators of the rural tourism cluster and the calculation of management projects. The cash flow model is not only a means of assessing the target function of the rural tourism cluster – the price of the rural tourism cluster – but also a tool of the enterprise management system. The latter eliminates the ambiguity of financial and personal factors in management and reduces the influence of the mentality of the manager-owner in the process of making strategic and operational decisions.

The quality of the management system depends significantly on the information that can be obtained by evaluating and processing the values of the parameters of its economic activity. Because such information is inaccurate, rural tourism cluster management systems belong to the class of stochastic feedback management systems. This is due to the properties of both aggressive market environment and the shortcomings of information systems. The problem of management synthesis is solved as follows: elaboration of the economic-mathematical model of the rural tourism cluster in the form of differential equations; definition of boundary conditions and constraints, equations of observation parameters; substantiation of criteria of optimality of management and characteristics of casual stimuli.

Oriented sign graphs can serve as a basis for solving multi-component models of enterprise management systems. System factors are used as vertices, with arcs (arrows) indicating the effect of changing one factor by changing another. To formalize the model of cash flows, we display it in the form of an oriented graph (Fig. 1) in which each vertex corresponds to the corresponding state of existing or future assets and liabilities of the rural tourism cluster, and the edges (arcs), to the direction of their movement.

All S_i are expressed in monetary units, and the arcs of the graph are characterized by the intensity (ρ_{ij}) and speed (v_{ij}) of the transition of assets and liabilities from one state i to a certain state j:

S_1 – value of investments of shareholders of rural tourism cluster;

S_2 – liabilities of the rural tourism cluster in the loan capital market: $S_2 = S_{2k}$, $k = 1, 2, \ldots$;

S_3 – tax payments for the observation period: $S_3 = \{S_{3k}\}$, $k = 1, 2, \ldots$;

S_4 – the total amount of free cash available;

S_5 – temporarily frozen funds;

S_6 – current value (including liquidity) of assets: $S_5 = \{S_{5k}\}$, $k = 1, 2, \ldots$;

S_7 – total, during the observation, of the amount of losses caused by the decrease in liquidity of assets, as well as costs associated with the payment of wages and dividends,

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payment of non-production costs, repayment of interest on debt, etc.;

S_8 – current value of tangible and intangible assets (long-term assets, fixed assets, inventories, receivables, and the image of the rural tourism cluster): $S_8 = \{S_{8k}\}$, $k = 1,2, \ldots$;

S_0 – the amount of depreciated asset value.

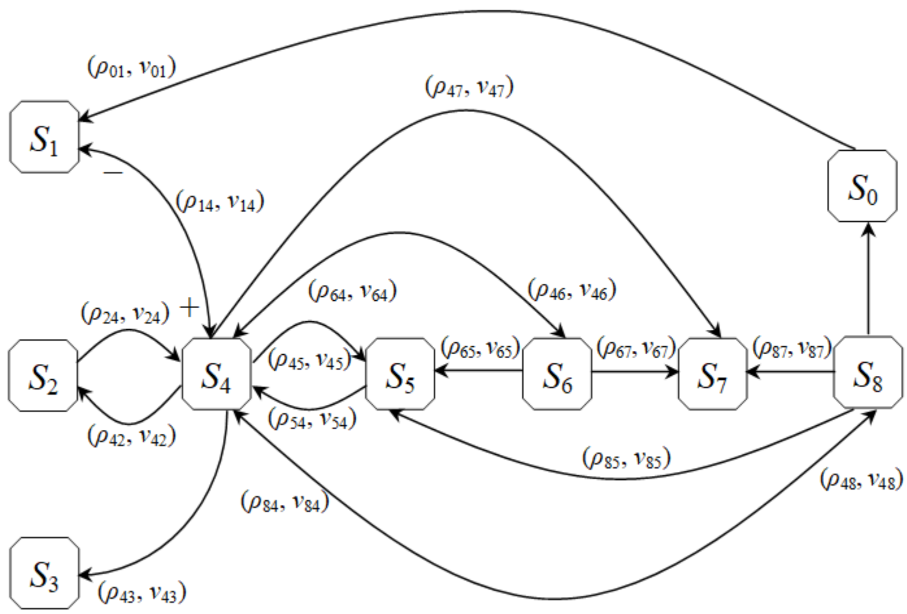

Figure 1. Directed sign graph of the cash flow model of the rural tourism cluster

**Source*: *developed by authors.*

It should be noted that between S_i, the intensity ρ and the rate v of transition of assets and liabilities from state i to state j, there is a functional relationship:

$$\rho_{ij} = \frac{v_{ij}}{S_i} \quad (1).$$

Results

To build an economic-mathematical model for optimizing the management of the parameters of the rural tourism cluster $S = \{S_i\}$, we use the method of estimating the state of a dynamic system based on observations (Kalman filter (Maybeck, 1979, Quinlan, 2009, Youngjoo, 2018)) and deterministic feedback control, which depends from the system state vector. Let $Y = \{y_i\}$, where $i = 1, 2, \ldots$ – the set of parameters of the rural tourism cluster, which are available for observation and are critical parameters of the system $S = \{S_i\}$, ie aggregated indicators of the rural tourism cluster, which characterize the dynamics of survival and development, in the sense of the target program of the system, as well as financial and economic indicators of the rural tourism cluster as a whole.

At some time interval $t \in (t_0, t_0 + \Delta t)$ the critical parameters of the rural tourism cluster are fixed, being related to the state of the rural tourism cluster $S = \{S_i\}$ by the following functional dependence (Maybeck, 1979):

$$Y(t) = \lambda_t \otimes S(t) + E_t \quad (2),$$

where E_t is the observation error (methodological error of the employee of the management staff, errors of economic analysis, etc.);

λt – is a given matrix of coefficients that are determined experimentally.

Then the transition of the system from a given state to a new one is determined by a system of differential equations

$$\frac{\partial S(t)}{\partial t} = \alpha_t \otimes S(t) + \beta_t \otimes C(t) + E_0 \quad (3),$$

where $C(t)$ is the vector of control influence, which is a function of the critical parameters of the system $Y = \{Y_i\}$, which determines the magnitude of the necessary changes of individual parameters of the system, to transition the system to a new state according to the target program of the rural tourism cluster;

α_t, β_t – given matrices of coefficients, which are determined experimentally;

E_0 – fluctuations caused by the influence of the external environment and/or errors of the control apparatus, leading to deviations from the target program of the rural tourism cluster.

The desired optimal vector of the control effect $C(t)$ is found using the method of estimating the parameters of the state of the dynamic system based on the results of observation from the minimizing quadratic criterion of control optimality:

$$\Omega = S^T_{t_0+\Delta t} \otimes A(t_0, t_0+\Delta t) \otimes S_{t_0+\Delta t} + \int_{t_0}^{t_0+\Delta t} \left[S_t^T \otimes B(t) \otimes S_t + C_t^T \otimes D(t) \otimes C_t \right] dt \quad (4),$$

where A, B – non-negatively given matrices, determined experimentally under the condition of minimal influence of fluctuations on the criterion of optimality of the control influence in comparison with the previous state of the system;

D – positive definite matrix, determined experimentally under the condition of minimal influence of fluctuations on the criterion of optimality of the control influence in comparison with the previous state of the system;

The optimal vector of control influence has the following form:

$$C_t^{opt} = -L(t) \otimes \left[\tilde{S}_t - F(t, \Delta t) \otimes S^{dp} \right] \quad (5),$$

S^{dp} – target program of the rural tourism cluster;

$F(t, \Delta t)$ – solution of the symmetric Riccati differential equation (Maybeck, 1979):

$$\frac{\partial F}{\partial t} = -F \otimes \alpha - \alpha^T \otimes F + F \otimes \beta \otimes D^{-1} \otimes \beta^T \otimes F - B$$

under the boundary condition $F(t_0 + \Delta t) = 0$;

$$F(t) = e^{\alpha_t * \Delta t} = 1 + \alpha_t \Delta t + \frac{\alpha_t^2 \Delta t^2}{2!} + \ldots + \frac{\alpha_t^n \Delta t^n}{n!} + \ldots$$

$\tilde{S}$ – estimation of the state vector of the system $S = \{S_i\}$, which are the solutions of the matrix differential equation:

$$\frac{\partial \tilde{S}_t}{\partial t} = \alpha_t \otimes \tilde{S}_t - \beta_t \otimes C_t + \Psi_0(t) \otimes \left[C_t - \lambda_t \otimes \tilde{S}_t \right] \quad (6),$$

where Ψ_0 is the matrix of Kalman-Bussey filter coefficients;

$L(t)$ – feedback gain:

$$L(t) = \beta_t^T \otimes E_{\tilde{S}}(t) \quad (7),$$

$E_{\tilde{S}}(t)$ – estimation error covariance matrix;

Thus, the control optimization algorithm using the target program is reduced to the following steps (Fig. 2):

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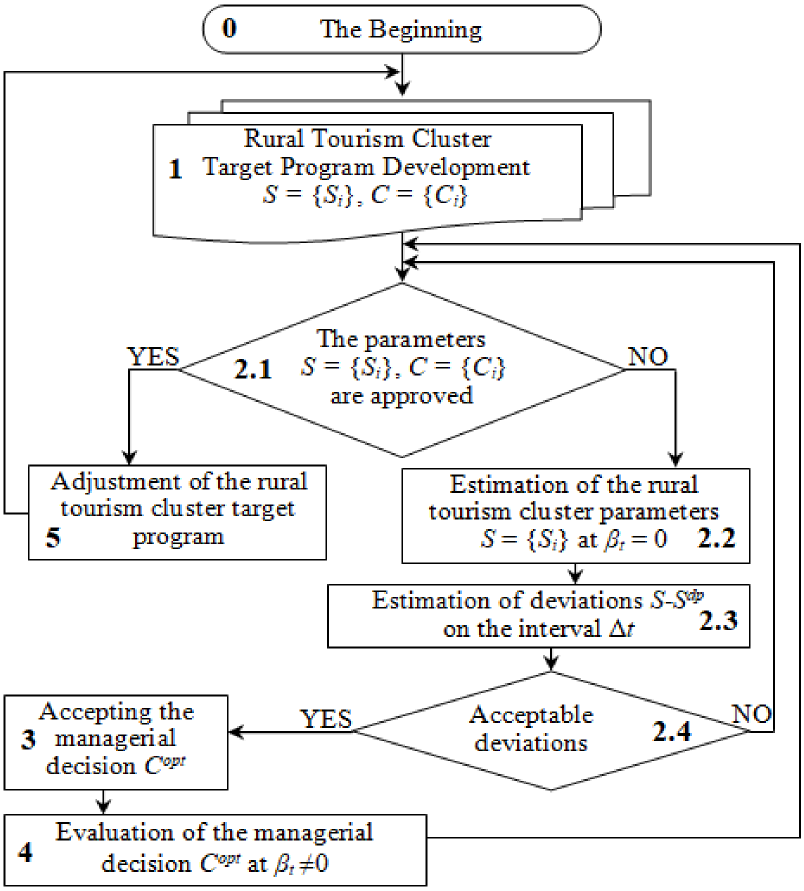

Figure 2. Algorithm for optimizing rural tourism cluster management

**Source*: developed by authors.*

1. Rural tourism cluster target program. Based on the method of program-target planning and economic-mathematical model of management apparatus optimization, admissible variants of rural tourism cluster target program development are developed (new state model – critical parameters of rural tourism cluster state $S = \{S_i\}$ and control influence vector $C = \{C_i\}$);

2. Rural tourism cluster target program optimization:

2.1. If the management of the rural tourism cluster approves the rural tourism cluster target program (parameters $S = \{S_i\}$ and $C = \{C_i\}$), then the management staff begins its implementation, with the subsequent transition to point 5;

2.2. In the time interval Δt in accordance with equation (7) to estimate the critical state parameters in the absence of initial conditions of the rural tourism cluster target program, ie the matrix β_t is zero-matrix;

2.3. On the time interval Δt the deviation of parameters of rural tourism cluster from the rural tourism cluster target program is estimated: $S – S^{dp}$;

2.4. If at any point in the time interval Δt the deviations have reached unacceptable values, then there is a transition to point 3, otherwise – a transition to point 2.

3. Synthesis of management decision ($C = \{C_i\}$). For time t, between Δt and the rural tourism cluster target program (S^{dp}) by formula (5) determine the desired control effect C^{opt}, which provides the maximum approximation of the critical parameters of the rural tourism cluster to the rural tourism cluster target program and move to point 4.

4. Evaluation of management decisions. By formula (6) we calculate the estimate of the parameters of the state of the rural tourism cluster for C^{opt} at $\beta_t \neq 0$ and make the transition to point 2.

5. Rural tourism cluster target program adjustment. The adjustment of the parameters of the control effect after the evaluation of the parameters of the state of the rural tourism cluster at each time point in the interval Δt and the transition to point 1.

Taking into account the fact that the cash flow model describes the relationships of rural tourism cluster performance indicators that can be calculated from financial and economic reporting data (ie those indicators that are a subset (vector) of critical observation parameters), as well as the fact that this subset is complete enough to calculate the whole set of observation parameters, we can apply the economic-mathematical model and control optimization algorithm (2)-(7).

We can write a system of differential equations for the cash flow model according to the graph of the cash flow model:

$$\begin{cases} \frac{\partial S_0}{\partial t} = \rho_{80} \cdot S_8, \\ \frac{\partial S_1}{\partial t} = -\rho_{14} \cdot S_1 + \rho_{81} \cdot S_8, \\ \frac{\partial S_2}{\partial t} = -\rho_{24} \cdot S_2 + \rho_{42} \cdot S_4, \\ \frac{\partial S_3}{\partial t} = \rho_{43} \cdot S_4, \\ \frac{\partial S_4}{\partial t} = \rho_{14} \cdot S_1 + \rho_{24} \cdot S_2 - (\rho_{42} + \rho_{43} + \rho_{45} + \\ + \rho_{46} + \rho_{47} + \rho_{48}) \cdot S_4 + \rho_{54} \cdot S_5 + \rho_{64} \cdot S_6 + \rho_{84} \cdot S_8, \\ \frac{\partial S_5}{\partial t} = \rho_{45} \cdot S_4 - \rho_{54} \cdot S_5 + \rho_{65} \cdot S_6 + \rho_{85} \cdot S_8, \\ \frac{\partial S_6}{\partial t} = \rho_{46} \cdot S_4 - (\rho_{54} + \rho_{65} + \rho_{67}) \cdot S_6, \\ \frac{\partial S_7}{\partial t} = \rho_{47} \cdot S_4 - \rho_{67} \cdot S_6 + \rho_{87} \cdot S_8, \\ \frac{\partial S_8}{\partial t} = \rho_{48} \cdot S_4 - (\rho_{80} + \rho_{81} + \rho_{84} + \rho_{85} + \rho_{87}) \cdot S_8 \end{cases} \quad (8),$$

where $S_i = S_i^T \otimes 1; I_j = I_i^T \otimes 1; 1 = \{1,1,...,1\}$ is a single column vector.

Based on the economic-mathematical model of management optimization and the cash flow model of the rural tourism cluster, it is possible to perform discrete enterprise management at time $t = t_0 + \Delta t$ by changing the values of the corresponding elements of the vector $V = \{v_{ij}\}$. Part of v_{ij} is within the operational control and depends on both the internal and/or external environments and the activities of the control apparatus. Given the relationship (1) between ρ_{ij}, v_{ij} and S_i, we obtain: $v_{ij} = \rho_{ij} \cdot S_i$.

We present system (9) taking into account the above conditions:

$$\begin{cases} \frac{\partial S_0}{\partial t} = \rho_{80} \cdot S_8, \\ \frac{\partial S_1}{\partial t} = -\nu_{14} + \nu_{81}, \\ \frac{\partial S_2}{\partial t} = \rho_{42} \cdot S_4 + \nu_{24}, \\ \frac{\partial S_3}{\partial t} = \rho_{43} \cdot S_4, \\ \frac{\partial S_4}{\partial t} = -(\rho_{42} + \rho_{43} + \rho_{45} + \rho_{47}) \cdot S_4 + \rho_{54} \cdot S_5 + \\ + \rho_{64} \cdot S_6 + \nu_{14} + \nu_{24} + \nu_{46} - \nu_{48}, \\ \frac{\partial S_5}{\partial t} = -\rho_{54} \cdot S_5 + \rho_{65} \cdot S_6 + \rho_{85} \cdot S_8 + \nu_{45}, \\ \frac{\partial S_6}{\partial t} = -(\rho_{64} + \rho_{65} + \rho_{67}) \cdot S_6 + \nu_{46}, \\ \frac{\partial S_7}{\partial t} = \rho_{67} \cdot S_6 + \rho_{87} \cdot S_8 + \nu_{47}, \\ \frac{\partial S_8}{\partial t} = -(\rho_{80} + \rho_{81} + \rho_{84} + \rho_{85} + \rho_{87}) \cdot S_8 + \nu_{48} \end{cases} \quad (9).$$

Since it is impossible to make unambiguous recommendations for the initial selection of the target program of the rural tourism cluster (item 1 of the management optimization algorithm), the initial matrices of model coefficients (2)-(7) are chosen so that the initial errors have less effect on the optimality criterion (4) and moments of time (Yeleiko et.al, 2016). For the rural tourism cluster cash flow model, we recommend making the following assumptions:

Let the observation vector Y coincide with the state vector of the rural tourism cluster S ($\{Y_i\} = \{S_i\}$); the matrix λ_t of equation (2) is a unit matrix ($\lambda_{ij} = 1$); all elements of the matrix B are equal to 0, except for the lower right (in the case of the cash flow model $b_{88} \neq 0$). In the general case:

A: $$a_{ij} = \begin{cases} \max\limits_{t \in [t_0, t_0 + \Delta t]} \{S_i^2\}, i = j \\ 0, i \neq j \end{cases} \quad (10),$$

B $$b_{ij} = \begin{cases} \Delta t \times \max\limits_{t \in [t_0, t_0 + \Delta t]} \{S_i^2\}, i = j \\ 0, i \neq j \end{cases} \quad (11),$$

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D: $$d_{ij} = \begin{cases} \Delta t \times \max\limits_{t \in [t_0, t_0 + \Delta t]} \{C_i^2\}, i = j \\ 0, i \neq j \end{cases} \quad (12),$$

Matrices $\beta_t = E_0$ in equation (3) are unit matrices.

Then equation (3) in vector-matrix form we write:

$$\frac{\partial S}{\partial t} = \alpha \otimes S + \beta \otimes C \quad (13),$$

where the matrix α and the vector C are determined from the system of differential equations (9):

$$\alpha = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{80} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{42} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{43} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\rho_{42}+\rho_{43}+\rho_{45}+\rho_{47}) & \rho_{54} & \rho_{64} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{54} & \rho_{65} & 0 & \rho_{85} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{64}+\rho_{65}+\rho_{67} & 0 & \rho_{87} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\rho_{80}+\rho_{81}+\rho_{84}+\rho_{85}+\rho_{87}) \end{pmatrix} \quad (14),$$

$$C^T = \left\{ \begin{matrix} 0; -v_{14} + v_{81}; -v_{24}; v_{14} + \\ + v_{24} - v_{48} - v_{46}; v_{45}; v_{46}; v_{47}; v_{48} \end{matrix} \right\} \quad (15).$$

Conclusions

The article discusses the significance of the DCF in evaluating the expected long-term income of a rural tourism cluster. It emphasizes that price is an integral criterion of management efficiency, while price maximization can be the target management function for the rural tourism cluster. The article determines that assets, liabilities and employees are significant parts of the cluster system of rural tourism. The concept of goodwill and how the system effect can be determined by comparing the total value of the system elements and the cost of the system overall by its target function are also addressed. Moreover, various factors influencing the value of the rural tourism cluster and its evaluation methods are considered.

The article proves that the critical parameters of the rural tourism cluster are recorded in a certain time interval and connected with the state of the rural tourism cluster by functional dependence. The transition of the system from a given state to a new one is determined by a system of differential equations. The sought optimal vector of managerial impact is found by the method of evaluation of parameters of the state of the dynamic system based on the results of observations by the minimizing quadratic criterion of control optimality. The algorithm of management optimization is reduced to certain steps based on a target program.

Therefore, the matrix α and the vector of control influence C unambiguously and with a sufficient degree of detail determine the space of the new state of the rural tourism cluster and the control influence that must be exercised to transfer the company from the previous state to a new one. The peculiarity of the presented economic-mathematical model and management optimization algorithm is that they can be applied to include operational and strategic and tactical decisions of enterprise management. This way of making management decisions can significantly reduce the influence of management staff on the effectiveness of these decisions by narrowing powers.

The developed economic-mathematical model and algorithm of management optimization are an integral part of the management system of the rural tourism cluster built on the basis of the system approach and serve as an effective tool. The implementation of such tools requires special knowledge and skills, as well as high awareness of managers and mastery of all indicators of economic activity. It is expedient to carry out such a model provided that the rural tourism cluster has a corporate automated management information system in which it will be possible to integrate a software module for the implementation of the economic-mathematical model and management optimization algorithm.

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