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Original paper

A power function profile of a ski jumping in-run hill

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The aim of the research was to find a function of the curvilinear segment profile which could make possible to avoid an instantaneous increasing of a curvature and to replace a circle arc segment on the in-run of a ski jump without any correction of the angles of inclination and the length of the straight-line segments. The methods of analytical geometry and trigonometry were used to calculate an optimal in-run hill profile. There were two fundamental conditions of the model: smooth borders between a curvilinear segment and straight-line segments of an in-run hill and concave of the curvilinear segment. Within the framework of this model, the problem has been solved with a reasonable precision. Four functions of a curvilinear segment profile of the in-run hill were investigated: circle arc, inclined quadratic parabola, inclined cubic parabola, and power function. The application of a power function to the in-run profile satisfies equal conditions for replacing a circle arc segment. Geometrical parameters of 38 modern ski jumps were investigated using the methods proposed.

Key words: *ski jumping, ski jump, in-run hill profile, mathematical modelling*

1. Introduction

There are four phases of a ski jumping: in-run, take-off, flight, and landing. According to the International Ski Competition Rules [6], only two last phases are taken into consideration while competition result is evaluated by judges. They evaluate a technique of a flight and landing and measure a length of a jump. The in-run and take-off (as previous phases) affect the finish phases, therefore they are the key phases which predetermine in a great part a sport result as a basis of quantitative and qualitative parameters of the jump [12].

A ski jumper executes the in-run and take-off sliding down the in-run hill which is ended with a take-off platform. The prevailing parts of well-known ski jumps are equipped with an artificial construction in the form of a solid unit which consists of the in-run hill and the take-off platform (figure 1). The profile of the in-run track includes three segments: two of them are straight-lines, and the third is curvilinear. The first straight-line segment *BC* and the curvilinear segment *CD* serve together actually as an in-run track. Another straight-line segment *DE* serves as a take-off platform. In a full sense, because of some inclination, the take-off platform serves as an in-run track as well. The inclination of the curvilinear segment at the highest point *C* is equal to the inclination of the first straight-line segment and at the lowest point *D* it is equal to the take-off platform inclination.

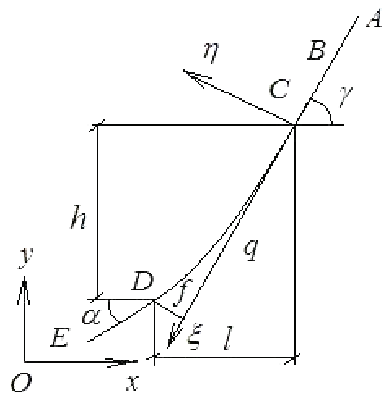

Fig. 1. An in-run hill scheme model

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Sliding down the curvilinear segment could be considered as a sub-phase of the in-run. The in-run curve starts when a ski jumper enters the radius and ends when he reaches the take-off platform. When a ski jumper enters a curvilinear segment, the normal reaction force increases due a centrifugal force. A take-off phase begins when a ski jumper initiates his take-off movement and ends just as he leaves the take-off platform.

According to the architecture norms, up to the present time, curvilinear segment has been constructed only as a circular arc [7]. Because an instantaneous increasing of the trajectory curvature on the border of the first straight-line segment and the circle arc, a ski jumper's body is affected by a centripetal force equal to about 87% of a normal reaction force value. Sliding along the arc, a skier's body is affected by a centripetal force which smoothly increases proportionally to the squared speed of sliding and disappears instantaneously at the border of the take-off platform. A normal force increases from 0.88 of the gravity on the first straight-line segment up to 1.65 on the arc [3]. The exact value depends on a slope, a speed, and a radius. During the straight-line motion, a normal force is less than gravity because a skier moves on a slope.

Aiming to control a reaction force when a ski jumper moves along a curvilinear segment, researchers propose to use profiles with a variable curvature: cycloid, parabola, inclined parabola, cubic parabola, hyperbola [9], and inclined cubic parabola [5]. The last one was presented by International Ski Federation (ISF) as a standard for the ski jump design. At any rate, one ski jump with inclined cubic parabola profile of the in-run hill in Bischofshofen, Austria, has been certificated by ISF [2].

There are various purposes and results while different profiles are proposed to be applied: to reduce a reaction force at the end of the curvilinear segment, or to stabilize its value, or to reduce its value to zero and to smooth the increasing of a centripetal force at the very beginning of the curvilinear track, and so on. However, the replacing of a circular arc with another profile function brings about an essential changing of some in-run hill parameters: the angle of a straight-line segment inclination [8] or horizontal and vertical dimensions of the curvilinear segment [1].

With a purpose to reduce the value of a normal reaction force just near a take-off platform, PALEJ & STRUK [9] considered cycloid, parabolic, and hyperbolic profiles and defined the first one of them as the best from this point of view. The authors formulated and solved an initial value problem for a nonlinear second-order equation. They characterized this approach to the problem as the simplest one and put emphasis on its disadvantage because the normal reaction does not usually appear at the point bordering with the take-off platform.

The researches tried to achieve the purpose of a decrease in a normal reaction force at the end of the curvilinear segment using a family of even polynomial functions which possessed the determined properties of the normal reaction [10]. Considering a popular K125 power ski jump Wielka Krokiew in Zakopane, Poland, they proposed to replace its straight-line and circle arc segments of the in-run hill with one polynomial curve. But a proper implication of this function carries the necessity to increase the inclination angle of the in-run hill in order to avoid the appearance of inflexion points. Unfortunately, the value of the increased incline should be greater than the maximum inclination of in-run hills of modern ski jumps [4].

The aim of our research was to find a function of the curvilinear segment profile which could make it possible to avoid an instantaneous increasing of a curvature and to replace a circle arc segment on the in-run of a ski jump without any correction of the angles of inclination and the length of the straight-line segments.

2. Materials and methods

Geometrical parameters of 38 modern ski jumps certificated by ISF were borrowed from the official web site of the Federation [6]. These parameters were: two angles of inclination of the straight line segments and a radius of the arc segment (table 1).

The methods of analytical geometry and trigonometry were used to calculate an optimal in-run hill profile. A simple geometrical model of the profile function was applied. A weak spot of the well-known models [3], [4], [8]–[10] takes into account an air drag force and a force of friction between skis and an in-run hill track. Corresponding models include empirical coefficients which depend of a skier's body position, a speed, a normal reaction force, temperature, dampness and others. Because analytical functions which have been used to model these factors do not ensure a corresponding precision, it is better to consider a problem of profile modelling passing over the drag and friction forces. It is not to ignore these forces, but to create a profile model without the necessity of taking them into consideration. Therefore, from a practical point of view, we considered the problem using a geometrical model [1]. There were two fundamental conditions of modelling: smooth borders between a curvilinear segment and straight-line segments of an in-run hill and concave of the curvilinear segment. Within the framework of this model, the problem has been solved with a reasonable precision.

Table 1. In-run hill parameters of ski jumps which are certificated by ISF [2]

No.	Locality (country)	Size K	γ	α	r
			Degree		m
1	Villach (AUT)	60	29.0	10.5	65
2	Wernigerode (GER)	63	35.0	9.5	59
3	Bischofsgrün (GER)	64	35.0	10.5	67
4	Namsos (NOR)	65	34.0	10.0	57
5	Bischofshofen (AUT)	65	35.0	10.0	65
6	Høydalsmo (NOR)	85	32.0	10.5	80
7	Villach (AUT)	90	35.0	10.5	64
8	Stryn (NOR)	90	30.0	10.5	85
9	Trondheim (NOR)	90	34.0	11.0	90
10	Örnsköldsvik (SWE)	90	36.0	10.5	90
11	Gällivare (SWE)	90	34.0	10.5	95
12	Heddal (NOR)	90	32.5	10.5	80
13	Mo I Rana (NOR)	90	36.5	10.5	80
14	Lillehammer (NOR)	90	35.0	11.2	90
15	Seefeld (AUT)	90	34.9	11.0	72
16	Lauscha (GER)	92	37.0	10.5	83
17	Oberwiesenthal (GER)	95	37.0	10.0	85
18	Hinterzarten (GER)	95	35.2	11.2	75
19	Gallio/Asiago (ITA)	95	30.0	11.0	90
20	Pragelato (ITA)	95	35.0	11.0	92
21	Sollefteå (SWE)	107	35.0	11.0	95
22	Ruhpolding (GER)	115	34.0	10.5	92
23	Zakopane (POL)	120	35.0	10.5	100
24	Engelberg (SUI)	120	35.0	10.5	110
25	Kuopio (FIN)	120	35.0	11.0	95
26	Kuusamo (FIN)	120	35.0	11.5	103
27	Trondheim (NOR)	120	34.0	11.0	105
28	Lillehammer (NOR)	120	34.0	11.0	107
29	Bischofshofen (AUT)	125	27.0	11.0	*
30	Klingenthal (GER)	125	35.0	11.0	105
31	Pragelato (ITA)	125	35.0	11.0	105
32	Whistler (CAN)	125	35.0	11.2	100
33	Garmisch-Partenkirchen (GER)	125	35.0	11.0	103
34	Willingen (GER)	130	35.0	11.0	105
35	Bad Mitterndorf (AUT)	185	35.0	10.7	147
36	Oberstdorf (GER)	185	39.0	10.5	120
37	Planica (SLO)	185	38.5	10.3	100
38	Vikersund (NOR)	185	40.4	11.0	105
Max		185	40.4	11.5	147.0
Min		60	27.0	9.5	57.0
M		108.4	34.6	10.7	90.8
SD		33.2	2.5	0.4	18.3

* Inclined cubic parabola.

3. Results

Four functions of a curvilinear segment profile of the in-run hill were investigated: circle arc, inclined quadratic parabola, inclined cubic parabola, and power function.

3.1. Circle arc profile model

According to the conditions of smooth joints between a circle arc and a straight-line segments, their angles of inclination should depend on horizontal ($l = x_C - x_D$) and vertical ($h = y_C - y_D$) dimensions of the curvilinear segment represented by the following equations (see figure 1):

$$l = r(\sin\gamma - \sin\alpha)\,; \quad h = r(\cos\alpha - \cos\gamma)\,, \qquad (1)$$

where:

r is a radius of the circle arc,

α is an angle of inclination of the second straight-line segment, i.e., take-off platform,

γ is an angle of inclination of the first one, i.e., an in-run hill, straight-line segment.

Based on equations (1) a ratio between the circle arc dimensions can be given by

$$\left(\frac{h}{l}\right)_{\text{circle}} = \tan\frac{\alpha+\gamma}{2}\,. \qquad (2)$$

A length of the circle arc is:

$$S = r(\gamma - \alpha)\,. \qquad (3)$$

Using equations (1)–(3) and the angles of inclination of straight-line segments (α, γ), we can calculate three of the four parameters of the arc segment (l, h, r, S), while priority should be given to one of them.

A circle arc profile has a deficiency. Because of an instantaneous increasing of the trajectory curvature on the border with the first straight-line segment, a ski jumper's body is affected by a centripetal force that has a commensurable magnitude with a body weight. A corresponding centripetal acceleration at the moment of entering to the circle arc (the point C in figure 1) is:

$$a_C = \frac{v_C^2}{r}\,, \qquad (4)$$

where v_C stands for a speed of sliding in the very beginning of the curve.

The curves representing the dimensionless value of the circle arc curvature

$$\frac{q}{r} = \sin(\gamma - \alpha)$$

and the profile relative to a longitudinal dimension

$$\frac{\eta}{f} = \frac{1 - \sqrt{1 - \left(\frac{\xi}{q}\right)\sin^2(\gamma - \alpha)}}{1 - \cos(\gamma - \alpha)}$$

of the in-run hill curvilinear segment with the angles of inclination α = 11° and γ = 35° are presented in figure 2. These parameters were used because of among 38 ski jumps certificated by ISF [2] seven have the same angles of inclination of the in-run hill, and another five – close to them, with the difference of ±0.2° (see table 1, numbers 20, 21, 25, 30, 31, 33, 34, and 9, 14, 15, 18, 32). These twelve ski jumps present a full range of the power (K = 90÷185) for high-level competitions in ski jumping. A ratio between the circle arc dimensions (2) is $(h/l)_{\text{circle}}$ = 0.424, and a dimensionless value of the curvature is $(l/r)_{\text{circle}}$ = 0.383.

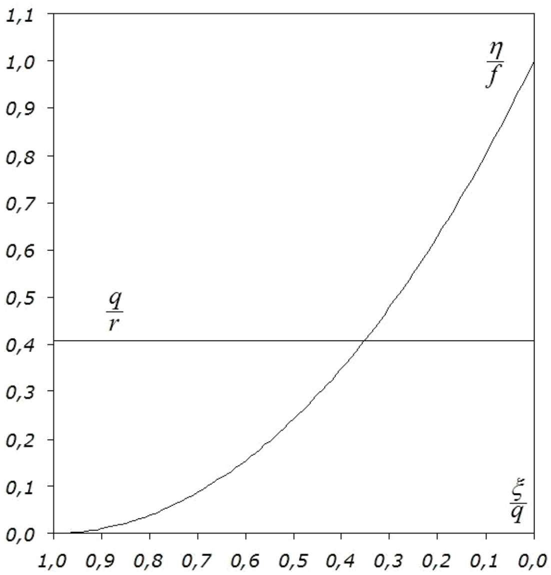

Fig. 2. Curves representing the dimensionless value of the circle arc curvature $\left(\frac{q}{r}\right)$ and the arc profile $\left(\frac{\eta}{f}\right)$ relative to a longitudinal dimension $\left(\frac{\xi}{q}\right)$ of the in-run hill curvilinear segment at angles of inclination α = 11° and γ = 35°

3.2. Power functions' profile models

These functions were relatively well defined in a rectangular coordinate system $\xi C \eta$ with the $C\xi$-axis as the continuation of an inclined straight-line in-run hill segment (see figure 1):

$$\eta = k\xi^p, \tag{5}$$

where:
k is a scale coefficient,
$p > 1$ is a power.

To define these parameters we need only three border conditions because the first straight-line segment is a tangent to this curve at the point C $\left(\frac{d\eta}{d\xi} = 0\right)$:

$$\xi_D = q; \quad \eta_D = f, \quad \left(\frac{d\eta}{d\xi}\right)_D = \tan(\gamma - \alpha), \tag{6}$$

where q and f are respectively the longitudinal and transversal dimensions of the curvilinear segment relative to $\xi C \eta$ coordinates.

Writing a formula for tangent inclination to the $C\xi$-axis as a derivation of the function (5)

$$\frac{d\eta}{d\xi} = pk\xi^{p-1}, \tag{7}$$

we arrive at the expression for the ratio of dimensions using the third border condition (6):

$$\frac{f}{q} = \frac{\tan(\gamma - \alpha)}{p}. \tag{8}$$

Taking account of (8), we obtain the ratio of the curvilinear dimensions relative to a horizontal Ox-axis (see figure 1):

$$\frac{h}{l} = \frac{\tan\gamma - \frac{f}{q}}{1 + \frac{f}{q}\tan\gamma}. \tag{9}$$

The values of the last ratio allowing the replacement of a circle arc segment profile with a quadratic (p = 2) and a cubic (p = 3) inclined parabolas for 37 certificated ski jumps are presented in table 2. The inclined quadratic parabola profile represents the ratio of the dimensions 1.4–4.6% smaller than the arc ratio and the inclined cubic parabola of the dimensions 16.0–18.8% bigger than this ratio. It is reasonable to expect that a power function profile with a power $2 < p < 3$ meets the same ratio of dimensions as an arc has. Using equations (2), (8), (9), we derived a formula for the power for this hypothetical profile:

Table 2. Parameters of the modelling of the in-run profile

Ski jump*	Arc	Inclined parabolas		p	$\frac{r}{\rho_D}$ (%)	ρ_D (m)
		Quadratic	Cubic			
	h/l_{circle}	$\frac{h/l}{h/l_{\text{circle}}}$ (%)				
1	0.359	98.6	116.1	2.05	94.8	68.5
2	0.409	96.7	119.0	2.11	90.3	65.4
×	0.419	97.1	118.2	2.10	91.0	73.6
4	0.404	97.2	118.4	2.09	91.4	62.4
5	0.414	96.9	118.6	2.10	90.6	71.7
6	0.389	98.0	117.3	2.07	93.0	86.0
8	0.369	98.4	116.6	2.06	94.3	90.2
9	0.414	97.6	117.5	2.09	92.1	97.8
10	0.430	96.8	118.4	2.11	90.3	99.7
11, 22	0.409	97.4	117.9	2.09	91.7	103.6
12	0.394	97.9	117.5	2.08	92.7	86.3
13	0.435	96.7	118.5	2.11	89.9	89.0
14	0.427	97.4	117.6	2.09	91.5	98.4
15	0.423	97.4	117.7	2.09	91.4	78.8
16	0.440	96.5	118.6	2.12	89.5	92.7
17	0.435	96.2	119.0	2.12	89.1	95.4
18	0.429	97.4	117.6	2.09	91.4	82.1
19	0.374	98.6	116.0	2.06	94.8	95.0
+	0.424	97.3	117.8	2.09	91.4	100.7
25	0.430	97.3	117.8	2.09	91.4	104.0
26	0.414	97.5	117.3	2.09	91.7	112.3
27, 28	0.344	97.6	117.5	2.09	92.1	116.2
32	0.424	97.4	117.6	2.09	91.5	109.3
35	0.421	97.2	118.0	2.10	91.1	161.3
36	0.461	95.7	119.0	2.14	87.9	136.5
37	0.454	95.8	119.0	2.13	88.1	113.5
38	0.481	95.4	118.8	2.15	87.1	120.5

* See table 1; × 3, 7, 23, 24; + 20, 21, 30, 31, 33, 34.

$$p = \tan(\gamma - \alpha)\frac{1 + \tan\gamma \times \tan\frac{\gamma+\alpha}{2}}{\tan\gamma - \tan\frac{\gamma+\alpha}{2}}. \qquad (10)$$

A value of power for the replacing of an arc with a suitable parabola in the case of the 37 certified ski jumps ranges from 2.05 to 2.15 (see table 2). A curvature at the end of the parabola equals 87.1–94.8% of the corresponding arc curvature.

A radius of curvature of the inclined parabola was calculated from the following formula:

$$\rho = \frac{\sqrt{\left[1+\left(\frac{d\eta}{d\xi}\right)^2\right]^3}}{\frac{d^2\eta}{d\xi^2}}. \qquad (11)$$

The curves representing the dimensionless values of this profile relative to a longitudinal direction of the in-run hill straight-line segment

$$\frac{\eta}{f} = \left(\frac{\xi}{q}\right)^p \qquad (12)$$

and its curvature

$$\frac{r}{\rho} = \frac{(p-1)\left(\frac{\xi}{q}\right)^{p-2}\cos^{-1}(\gamma-\alpha)}{\left[1+\left(\frac{\xi}{q}\right)^{2p-2}\tan^2(\gamma-\alpha)\right]^{\frac{3}{2}}} \qquad (13)$$

for the ski jumps with the angles of inclination $\alpha = 11°$ and $\gamma = 35°$ are presented in figure 3. As regards all the 38 ski jumps, the curvature at D point (see tables 1, 2) of the power function ($\rho_D = 62 \div 161$ m) is slightly smaller than that in the corresponding arc ($r = 57 \div 147$ m).

When a power is greater than 2 and the second derivation of function (5) equals zero at the top of the curvilinear segment (see figure 1, point C, $\xi = 0$), there is no centripetal acceleration according to equation (4): $\rho_{(\xi=0)} \to \infty$. Sliding down the curve, a skier's body smoothly increases its centripetal acceleration which reaches a maximum on a border of the take-off platform ($\xi = q$):

$$a_D = \frac{v_D^2}{\rho_D}, \tag{14}$$

or close to middle of the segment, where ρ_D is a radius of curvature at this instant; v_D is a corresponding skier's speed. This value is determined according to the conditions of safety for each ski jump [1].

4. Discussion

Using a cubic parabola profile for the ski jump with the parameters of the in-run hill ($\alpha = 11°$; $\gamma = 35°$), we can get nearly the same ratio of curvilinear segment dimensions (h/l) as that of a circle arc profile: a relative difference equals –0.7% [1]. After the cubic parabola the nearest (according to a modulus of the difference of a ratio with a circle profile) to this ratio are cycloid, inclined quadratic parabola, quadratic parabola, hyperbola, and inclined cubic parabola [5]. A maximum ratio of a curvilinear segment has an in-run hill profiled with an inclined cubic parabola, and a minimum – hyperbola.

Only a circle arc profile has a constant curvature, the other six functions, which have been considered as hypothetic profiles, have a variable curvature. The curvature increases down a hill when a curvilinear segment is profiled as a quadratic parabola and cubic parabola, and decreases when it is profiled as an inclined quadratic parabola, hyperbola, cycloid, and cubic parabola.

Using an inclined power function ($p > 2$) profile only, we can get a zero value of a centripetal force on a border of straight-line and curvilinear segments (the point C in figure 1). All the other functions investigated (circle, cycloid, hyperbola, quadratic parabola, inclined quadratic parabola, and cubic parabola profiles) could not assume a zero value because of an instantaneous increasing of the trajectory curvature on the border.

A circle arc hill of a ski jumps K185 ($\alpha = 10.5°$; $\gamma = 39°$) in Oberstdorf (GER) could be replaced by a cubic parabola profile with almost the same ratio of the curvilinear segment dimensions (a difference in ratios is –0.04%). A curvilinear segment of an in-run hill of a ski jump K125 ($\gamma = 27°$, $\alpha = 11°$) in Bischofshofen (AUT) is profiled with an inclined cubic parabola (see table 1, No. 29). If it were be profiled with a simple cubic parabola, a ratio of dimensions of an in-run hill curvilinear segment would be equal to 0.340, and if it were be profiled with an inclined quadratic parabola – a ratio would be equal to 0.395. If a curvilinear segment were be profiled with a circle arc – the ratio would be equal to 0.344.

Ski jumps of a similar size (K125) in Klingenthal (GER), Pragelato (ITA), and Garmisch-Partenkirchen (GER) have a curvilinear segment which is profiled with a circle arc (see table 1, Nos. 30, 31, 33). They have the same angle of inclination of a take-off platform ($\alpha = 11°$) but rather greater angle of inclination of a straight-line segment ($\gamma = 35°$). And their ratio between the dimensions of an in-run hill curvilinear segment is rather greater: 0.424. If a curvilinear segment of the in-run hill of these ski jumps is profiled with an inclined cubic parabola, the ratio will be equal to 0.500, and using a simple cubic parabola, we have $h/l = 0.421$.

PALEJ & STRUK [10] proposed to replace the straight-line (BC) and circle arc (CD) segments of an in-run hill with one curvilinear segment (BD) profiled as a polynomial of the second, fourth, sixth, and eighth power (see figure 1). The function was constructed on condition that a normal reaction affecting a skier's body on the curvilinear segment had a non-zero value. The function was calculated as a solution of a nonlinear differential equation of the second order. The authors declare a positive consequence of this reconstruction: reducing a curvature decreases a normal reaction of a skier's body. In general, in result of this replacing, a straight-line segment of the in-run hill does not disappear. It only becomes shorter up to a straight-line segment AB where a start gate is situated. Sometimes, but very seldom, a start gate can be placed at the point A, then according to this model, the in-run should be started at the very beginning of this curvilinear segment.

This model of an in-run hill construction has a few defects which make this approach a dubious one in a practical plane. Firstly, according to this method, the value of the increased incline should be greater than the maximum inclination of in-run hills of modern ski jumps: $\gamma = 29.0$–$40.4°$ (see table 1). Secondly, it is a doubtful reason for a decrease in a curvature of the in-run hill on a border with a take-off platform. A corresponding decreasing of a centripetal force results in the same decreasing of take-off impulse at the very beginning of the phase. Thirdly, because a dynamic problem was considered, an air drag and a ski friction were not taken into account within the framework of the model. There is a significant influence of these forces on the dynamics of a ski jumper's in-run [3].

In order to control an inertia force acting on a ski jumper's body during sliding down, PALEJ & FILIPOWSKA [8] proposed to replace the first straight-line segment and a circle arc segment with one curvilinear segment of a hypothetic profile in the form of a polynomial function. Aiming to avoid the appearance of inflexion points, they were forced to increase an angle of inclination of a start segment. In an example of a K120 ski jump (see table 1, No. 23) in Zakopane (POL), which was considered, the angle of inclination in a circle arc (γ = 35°) was increased up to 41°80′÷49°68′ correspondingly to a power of the polynomial that equalled 2÷8. If they used a cycloid, a quadratic parabola, and hyperbola [9] they would increase the angle of inclination up to 46°16′÷55°19′. These values are significantly greater than a standard value of the in-run hill inclination.

If a quadratic parabola or an inclined cubic parabola is applied to replace a circle in-run hill profile, their horizontal and vertical dimensions should be greater than the corresponding dimensions of a circle profile. A corrected length of a straight-line segment of an in-run hill should be smaller. If a cycloid, an inclined quadratic parabola, or a hyperbola profiles are applied, the dimensions should be smaller. A corresponding corrected length of a straight-line segment should be greater. If a cubic parabola profile is applied, its dimensions should be greater, smaller or equal to the circle dimensions, depending on the angles of inclination of an in-run hill and a take-off platform [1].

The only profile which obtains a zero centripetal acceleration at the top point of the curvilinear segment is an inclined cubic parabola and power function (see figure 3). Therefore, we considered in a special way the virtual replacing of a real circle arc profile of an in-run hill with an inclined cubic parabola profile. Because a K125 ski jump in Bischofshofen (AUT) has been originally designed with an inclined cubic parabola profile, the corresponding line No. 29 (see table 1) in table 2 was not completed. Aiming to equip the considered ski jumps with an inclined cubic parabola profile, we should increase a horizontal dimension by 32.4÷43.9%, and a vertical one – by 54.7÷65.2%. A relative (to the horizontal dimension) length of a straight-line segment of the in-run hill should be decreased by 42.6÷49.3% [1].

For example, a ski jump K120 (see table 1, No. 23) Wielka Krokiew in Zakopane (POL) could be reconstructed and equipped with an inclined cubic parabola profile instead of a circle arc profile by increasing its horizontal and vertical dimensions respectively by 37.3% (14.59 m) and 62.2% (10.22 m); a relative length of a straight-line segment should be decreased by 45.5% (17.81 m). According to PALEJ & STRUK's method [9] the dimensions should be increased respectively by 53.8% (21.07 m) and 89.9% (14.75 m); a relative length of a straight-line segment should be decreased by 65.7% (25.72 m).

There are three advantages of our method of the reconstruction of in-run hill. Firstly, an angle of inclination of hill remains the same. Secondly, there is no inflection of the curvilinear segment. Thirdly, a significantly smaller part of a straight-line segment should replaced with a curvilinear one.

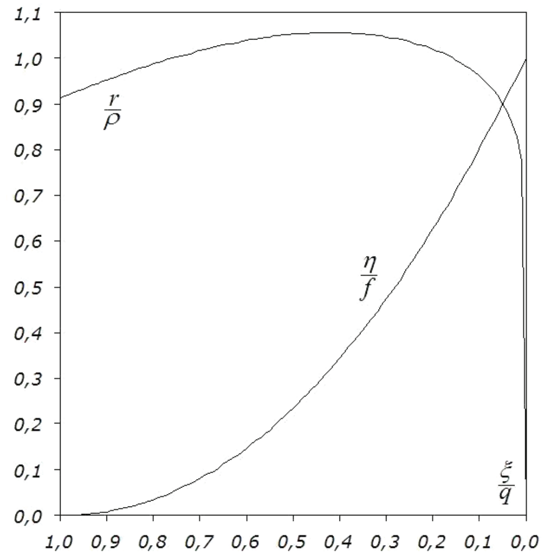

Fig. 3. Curves representing the dimensionless values of the parabola curvature $\left(\frac{r}{\rho}\right)$ and the corresponding profile $\left(\frac{\eta}{f}\right)$ relative to a longitudinal dimension $\left(\frac{\xi}{q}\right)$ of the in-run hill curvilinear segment with angles of inclination α = 11° and γ = 35°

5. Concluding remarks

The application of a power function to profile the curvilinear segment makes it possible to avoid an instantaneous increasing of a curvature and to replace a circle arc segment on the in-run of a ski jump without any correction of the angles of inclination and the length of the straight-line segments.

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